A Mathematical Analysis of the Structure of the Jewish Calendar:

Perfection as the Enemy of the Very Good

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Abstract

Much literature of the last hundred years has insisted that the Jewish calendar contains more than a six-and-a-half minute error which causes it to lose a day every 216 years. In fact, even a simple mathematical analysis of the underpinnings of the calendar shows that one cannot devise a luni-solar calendar without this (approximately) 6 2/3 minute gap unless one vastly expands the matrix of the calendar to 334 years and 4,131 lunar months, a matrix so large as to be unworkable. Thus, the nineteen-year calendar, like many other templates constructed by the Talmudic Sages (*Chazal*), strikes a balance between ease of use and accuracy.

Do not let these methods be light in your eyes just because we do not need them in our time, for these methods are distant and deep. This is the "secret of intercalation" that the great sages knew; they are not passed on to all people, only to ordained sages with understanding. But the calculations used in our times, when there is no high court to establish the months visually, the ones we use today, even young school children can master them entirely in three or four days. Perhaps one of the scholars from among the nations or Jewish scholars who have learned the Greek science will delve into these methods that I use to calculate the appearance of the new moon and see a slight approximation in some of the methods, and then think that this escaped me and that I was not aware that an approximation was used in this method. This should not enter his mind; rather, every thing in which we were inexact was because we knew fundamentally with clear geometric proof that this would not detract from knowing the appearance of the moon and therefore of no concern. So too, when you see in one of the methods a small amount missing from a calculation appropriate to that method, we have done this intentionally because there is a corresponding additional amount in another method so that the final outcome is approximated without long calculations. This is done so as not to confuse a person not practiced in these matters by a great many calculations, which are not truly necessary for calculating the appearance of the new moon.

Rambam, Kiddush Hachodesh 11:4-6

Le mieux est l'ennemi du bien.

Voltaire, La Bégueule

Much of the literature on the Jewish calendar in the last many years has focused on the slight discrepancy between the length of the solar year approximated in the 19-year cycle of the Jewish calendar and the more precise astronomical measurement currently available of the earth's rotation around the sun. Because of this disparity, the lunar and solar years of the Jewish calendar are not perfectly synchronized with the true astronomical solar

years; a discrepancy of one full day per 216 years creeps in—the famous six-and-two-thirds-minutes-per-year gap. This has led to Pesach occurring later and later in the spring over time.[1] This is indeed a problem, the literature maintains, of great magnitude and significance, is cause for concern, and—similar to the Julian calendar before the Gregorian reform—creates a pressing need for solution.

The Jewish Encyclopedia, for instance, calls attention to such issues in a bold, indented paragraph entitled "Error in the Calendar." It states:

> That there is a slight error in the Jewish calendar—due to inaccuracies in the length of both the lunar and the solar years upon which it is based—has been asserted by a number of writers .... The assumed duration of the solar year is 6 minutes, 39 25/57 seconds in excess of the true astronomical value, which will cause the dates of the commencement of future Jewish years, which are so calculated, to advance from the equinox a day in error in 216 years .... According to these calculations the Jewish year exceeds the Gregorian by 6 m. 39.37 s. and the Jewish month by .492 s. *Insignificant as these differences may appear, they will cause a considerable divergence in the relations between Nisan and spring as time goes on, and may require a Pan-Judaic Synod to adjust.*[2]

Similar observations are made by the more modern *Encyclopedia Judaica,* although a different remedy (no less

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[1] In the 11th Century CE, Pesach fell out at the earliest on March 17 and at the latest on April 15 (per the Julian calendar). In the 20th Century CE, the earliest Pesach fell out was March 26, and April 25 at the latest (Gregorian).

[2] "Calendar," Joseph Jacobs and Cyrus Adler, *The Jewish Encyclopedia* III: 300-301 (Funk and Wagnalls, NY, 1910) (emphasis added). This magnificent work was recently put online at www.JewishEncyclopedia.com.

radical than a pan-Judaic Synod) is suggested–the return to visual sanctification of the month.[3] *Encyclopedia Judaica* states:

> If the average length of the solar year in the present Jewish calendar exceeds [the modern estimate of the length of the solar year] by approximately 6 2/3 minutes, this discrepancy was left out of account as it was assumed that its cumulative effect would remain negligible over a long period at the end of which the present system was expected to be replaced again by a system based on true values more akin to the earlier Jewish calendar in which New Moons ... and intercalations were proclaimed on the basis of both observation and calculation.

Even the *Encyclopedia Britannica* makes note of the discrepancy (though with no remedy suggested).[4]

An underlying assumption by the calendar critics, either implicit or explicit, is that were the rabbinic approximation of the solar year more accurate, this error would not have crept in. By extension, had the rabbis used a more precise figure, a different—and presumably better—Jewish calendar could have been constructed.

This article will demonstrate that this assumption is mathematically incorrect: even if a more precise measurement of the solar year were to be incorporated into the Jewish calendar,

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[3] *Encyclopedia Judaica,* Vol. 5 (Jerusalem: Keter Publishing Co., 1971), Col. 47, s.v. "Calendar."

Even Dr. Feldman, in his classical work, calls this issue a 'defect;' see, W. M. Feldman, *Rabbinical Mathematics and Astronomy,* 3rd ed. (London: Herman Press, 1978), 207.

[4] See "Calendar," *Encyclopedia Britannica,* 2003, Encyclopedia Britannica Online, 30 June 2003 http://search.eb.com/eb/article?eu=108733 which states:

> The present Jewish calendar is mainly based on the more accurate value 365 days, five hours, 55 minutes, 25 25/57 seconds—in excess of the true tropical year by about six minutes, 40 seconds. Thus, it is advanced one day in about 228 [sic] years with regard to the equinox.

one would still choose to adopt a 19 year cycle, despite the slight "error" inherent to it. Were the calendar to select an alternative solution to minimize the disparity between lunar months and solar years, a much larger cycle would be necessary. It turns out that no better solution than our current 19-year/235-month cycle exists until you reach 334 years and 4131 months.[5] Such a system only gains one day every 28,524 years. However, this increased mathematical precision would come at a high cost. The calendar would give up its essential features—very simple patterns, portability, and ease of use. Even as simple a notion as a mnemonic device for remembering the seven leap years in the 19-year cycle would be lost; a 334-year cycle would require memorizing the positions of 123 leap years (albeit with some patterning)! Replacing the 19-year cycle with one of 334 years would not only strip the Jewish calendar of its wonderful elegance and simplicity, but also undermine the rabbinic goal of a practical, workable, universal Jewish calendar.

Indeed this article will underscore that the length of the solar year presumed by the Jewish calendar (often referred to as the seasons (*tekufot*) of Rav Adda) is a derived—rather than an observed—astronomical value. The number comes about from

[5] Though 182, 201, 220, 239, 258, 277, 296, and 315-year cycles also seem to produce better results than the 19-year cycle, upon closer examination one sees that these are just differences from 334 by multiples of nineteen (i.e., 334 minus 19x8, 19x7, 19x6, 19x5, 19x4, 19x3, 19x2, and 19x1, respectively). Consequently, the attendant gains in difference per cycle (standardized by year) improve as one approaches 334. Thus, for instance, a 182-year cycle deviates from 2251 months by 5.65 minutes per year (and loses a day every 255 years), while a 201-year cycle deviates from 2466 months by 4.48 minutes per year (and loses a day every 321 years), and so on. The most dramatic leap (9X) still occurs when moving from 315 to 334 solar years, with the 315-year cycle deviating from 3896 months by 0.455 seconds per year (losing a day every 3167 years) while the 334-year cycle loses only 3.023 seconds per year (losing a day every 28,524 years).

having lunar months as the basic building block of the Jewish calendar, and only secondarily having the lunar months correspond to the solar seasons (and solar years). The discrepancy between the length of the solar year according to the Jewish calendar and the actual astronomical solar year was not necessarily an error unknown to the rabbinical sages. Since no matter which measurement they started with, they would have chosen to create a calendar with a 19-year cycle, this error would have been an inevitable inclusion; greater accuracy in the solar years does nothing to fix this problem. One should thus view this choice as a conscious decision on the part of the calendar's creators to have a functional and accessible Jewish calendar for all Jews to use (meaning derive) easily. A more precise but fundamentally unusable calendar would have defeated their original intent.

Finally, this article will demonstrate that the solution offered by many to reconcile the Jewish solar year with the astronomical solar year caused by the loss of a day every 216 years is fundamentally flawed. The "simple" notion of one day having the Jewish calendar return to the hands of the Sanhedrin and oral testimony, thus enabling Jewish leaders to subtract days from the calendar so it can "catch up" to the solar cycle in fact creates more problems than it solves. **Once the Jewish calendar postulates that its basic building block is the lunar month, the months are established to correspond to the Molad (the lunar conjunction—related, but not identical to the appearance of the new moon). Subtracting a day may push the calendar closer to the current solar cycle, but it will break the faithful connection of Rosh Chodesh to the Molad date.** Another solution is indeed workable, but it cannot be implemented for many hundreds of years. It will be explained in the section entitled "Long Term Solutions to the Calendar Difficulties."

Introduction: Calendar Basics

Our concept of a year is based on the earth's rotation around the sun. Over that period of time, one sees the earth experience four different seasons. When one sees the sun return to its original position in the sky, the cycle of seasons begins anew. The amount of time it takes the earth to rotate around the sun is currently 365.242190 days.

Our concept of a month is based on the moon's rotation around the earth. Over the course of this cycle, the moon changes positions relative to the sun; thus one sees more or less of the surface of the moon reflecting the sun's light as the angle changes. We traditionally measure a month from new moon (commonly believed to be the time the first sliver ought appear, though in astronomical terms better expressed as mean lunar conjunction) to new moon. The cycle of waxing and waning currently repeats every 29.5305889 days. Twelve months approximately equal one year.

A calendar is simply a convention for calculating the length of a year and dividing it into workable parts. There are three basic models of calendars extant.

The first is the solar calendar. This system takes the earth's rotation around the sun in about 365 days as its primary unit of time. This structure ignores the measurable astronomical phenomenon of lunar months. It currently uses months merely as twelve administrative sub-parts of the calendar.[6] But it would be just as logical in this system that there be 10 months of 36 or 37 days as 12 months varying from 28 to 31 days. The secular

[6] Likely because the signs of the zodiac rotate 30 degrees (1/12 of 360) in 1/12 of a solar year. In 1931, the League of Nations proposed changing the calendar to have 13 months of between 28 and 29 days. For more on this, see e.g., W. M. Feldman, *Rabbinical Mathematics and Astronomy*, 3rd ed. (London: Herman Press, 1978), 208-209. (Like many activities of the League of Nations, nothing was accomplished.)

calendar adopted from the Catholic Church, as well as the Thermidorian calendar of the French Revolution, are solar calendars.

The second is a lunar calendar. This system's primary building block is the moon's rotation around the earth over the course of one month. This structure ignores the measurable astronomical phenomena of solar years. A year is just a rough approximation of the solar cycle, having 12 lunar months of 29 or 30 days on a rotating basis, and composed of 354 days—11 short of the solar cycle. Each lunar "year" thus falls farther and farther behind the solar calendar. The Islamic calendar follows this model.

The third possibility is a luni-solar calendar. This system has lunar months and attempts to reconcile them with the solar year as well. Unfortunately, the length of the solar year is not a multiple of the length of the lunar month. This means that with 12 months per year, the relationship between our solar year and our lunar month cannot be maintained in a single 12-month cycle. A luni-solar calendar relies on intercalation, the addition of "leap" months over the course of a number of cycles, in order to maintain a correspondence between the lunar months and solar years. The Hindu calendar, for instance, is a luni-solar calendar.

The Jewish Calendar

The Jewish calendar, too, is a combined luni-solar calendar. Its basic building block is the lunar month, following the biblical commandment to reckon new months. The Jewish calendar further strives to have its years keep up with the solar seasons and years, rather than "drift" farther behind. This, too, is explicitly dictated by the Torah. We are commanded to keep the festivals in their appropriate times of the year,[7] and we are

[7] Exodus 13:10

further enjoined to safeguard Pesach so that it occurs in the spring.[8] Though Rambam (Maimonides) identifies the obligation to reckon months and the requirement to follow the solar seasons as the same mitzvah, while Ramban sees them as two separate *mitzvot*, the obligation to create a luni-solar calendar is, according to all, a biblical mandate.[9]

Getting the lunar and solar cycles to correspond is a somewhat complicated goal. As discussed earlier, lunar months do not perfectly correspond to solar years. Periodic intercalation is necessary to keep the two cycles together. The Jewish calendar does so with a cycle of 235 months over 19 years, with seven leap months inserted periodically (19 multiplied by 12, plus seven, equals 235).

The 19-year cycle was known as far back as the period of the ancient Greeks. Meton of Athens (ca. 440 BCE) observed that 19 solar years are equal to 235 (actually 234.997) lunar months. Or, in arithmetic terms, 365.2421990 days per year multiplied by 19 solar years is approximately 29.53059375 days per month multiplied by 235 (actually 234.997) lunar months. [10] (The 19-year cycle is thus also known as the Metonic cycle.) So every 19 years the phases of the moon fall on the same solar dates.[11] The rabbinic sages adopted the 19-year cycle to construct the Jewish calendar. They took the lunar month as their primary building block. In order to

[8] Deuteronomy 16:1

[9] See Rambam, *Sefer Hamitzvot, Mitzvot Aseh* 153, and Ramban *ad loc.*

[10] This can also be calculated in reverse by asking what whole integer when multiplied by 365.2421990 (a solar year) is equal to 29.53059375 (a lunar month) multiplied by what other whole integer. Nineteen and 235, respectively, are the closest reasonable answers, as they generate a difference of only .003 months from the closest whole month.

[11] This is not entirely correct, as leap days must be added to the solar calendar to account for the fractional remainder of the earth's rotation around the sun that exceeds 365 days. (Currently 97 leap days per 400 years are added to the solar calendar.) More correct would be to say that every 19 years the phases of the moon recur when the sun is approximately in the same position in the sky.

have 235 lunar months in 19 years, one has to have 12 years of 12 lunar months and seven years of 13 lunar months in the Jewish luni-solar calendar.

So, to summarize: There are four important numbers to note:

1] A solar year is 365.242199 days.
2] A lunar month is 29.53059375 days.
3] Nineteen solar years are 235 lunar months.
4] In order to have 235 lunar months in 19 years, one has to seven lunar leap months in the 19 year cycle (as 12 months multiplied by 12 years, plus 13 months multiplied by seven years, equals 235).

Mathematical Calculations of the Jewish Calendar

Through the Second Commonwealth period and the Tannaitic period, the Jewish calendar was set on a visual basis. The Sanhedrin took testimony about when the new moon was seen every month and pronounced the months accordingly. After the new month was declared, the information needed to be disseminated throughout the Jewish community. The central leadership also kept the lunar cycle consonant to the solar years. Leap months were added on an irregular basis when climate conditions indicated that spring had not yet arrived.

By the middle of the Talmudic period, though, Jewry switched to a calculated new-moon and solar year. All Jews around the globe have used this system for at least the last 1600 years. When exactly the calculated calendar began is a matter of some dispute. While Rambam maintains that this took place prior to the time of the Mishnah, most authorities are of the opinion that the calendar was set in the time of Hillel II, around 359 CE.[12]

[12] Rambam nevertheless maintains that visual sanctification continued through the period of Abayee and Rava (*Kiddush Hachodesh*

As ever, the basic building block of the Jewish calendar remained the lunar month. Following the tradition of Rabban Gamliel, the new moon was presumed to be 29 days, 12 hours and 793 parts[13] (44.055555 minutes) from the last moon, for a calculation of 29.53059413 days per lunar month.

Knowing the Molad of one month allowed for following months to be easily calculated. The remainders beyond even weeks (multiples of seven days) were carried through the year. One iteration would give the beginning of the next month. Twelve cycles from the previous Tishrei would indicate the date of the following Rosh Hashanah. Leap months were to be added on a regular basis in years three, six, eight, 11, 14, 17, and 19 of the 19-year cycle. Since the 19-year cycles were understood to correspond to 235 months, at the end of a cycle, the remainders from all the Moladot would become a standard figure per cycle. Thus with just a few pieces of information—the Molad of the Creation year, the remainder for each Molad, and which year (and month) of the 19-year cycle it is—one can easily derive the dates and observances of the Jewish calendar as well as carry them well into the future.[14]

The leap months were inserted, as stipulated earlier, to prevent the lunar cycles from falling behind the solar seasons. To this end, the solar year was taken to be 365 days, five hours and

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5:2-3). A full discussion of the history of the Jewish calendar is beyond the scope of this article. For a full treatment of the history, see *Encyclopedia Judaica*, ibid., and J. David Bleich, *Bircas Hachammah: Blessing of the Sun—Renewal of Creation* (Mesorah Publications Ltd., 1980).

[13] An hour is subdivided into 1080 "parts", *chalakim*. Each *chelek* is thus 3 1/3 seconds. Since 793 is a prime number, 793 forces the use of 1080 as the counting system instead of seconds, as it is the smallest whole number that can be used to express the percent of the hour in the lunar rotation ratio of 793/1080.

[14] For concise instructions on the methods for deriving the Jewish calendar, see remarks of *Mishnah Berurah* at the end of the fourth volume.

997.63 parts (55.4239 minutes) long, for a calculation of 365.246822 days in the solar year. It is important to note that though this value for the length of the solar year, attributed to Rav Adda, is used as an assumed value (input) when calculating the length of solar seasons (*tekufot*), it is in truth a derived average value—1/19 of 235 lunations—rather than an actual measurement.[15] This correlates to our original understanding of the Jewish calendar – that it is primarily based on lunar months and only secondarily keeps up with the solar cycle.

Difficulties of the Calendar

So far, we have seen the mathematical underpinnings of the Jewish calendar that allow one, with very little data, to create a fully functional calendar independent of place or authority. A problem arises, though, because the lengths of time assumed by the Jewish calendar are not 100 percent accurate. The lunar cycle is off by only a minute amount, 0.00000485,[16] creating practically negligible differences over time.[17] The length of the solar year, however, is off by 120.8 *chalakim* (six minutes and 39 seconds per year), as the astronomically correct calculation of the solar year is 365 days, five hours and 48.7666 minutes (877.8 *chalakim*). This

[15] Indeed, when Rambam calculates the difference between the lunar and solar year, he uses 365.25 as the length of the solar year (*Kiddush Hachodesh* 6:4).

[16] Because of gravitational and other astronomical changes, the lunar cycle has changed slightly over the last centuries. It is now 29.5305889 days, but it has varied. Around 1900 its length was 29.5305886 days, and around 2100 it will be 29.5305891 days. Judaism uses the established number of 29.53059375. The Solar year also varies slightly based on astronomical changes. Its length is currently 365.242190 days, but it varies. Around 1900 its length was 365.242196 days, and around 2100 it will be 365.242184 days.

[17] 12.37449796 seconds (0.00000485 * 29.5305889 * 24 * 60 * 60) per year, or one day every 6982 years (12.374498 seconds times 6982 years equals 86399 seconds, which is 23 hours, 59 minutes, and 59 seconds).

means that every year the Jewish calendar runs about 6.65 minutes ahead of the solar year and the equinoxes. After 216 years the Jewish calendar is a full day past the solar calendar,[18] and after 1500 years it is seven days ahead. This explains why Pesach is occurring later in the solar year in the last two centuries than it did in previous centuries.[19]

To see if there is a better solution than 19 years and 235 months, one can tabulate (see Chart 1) the years in a solar cycle from one to 1,002 (shown in part in column A) and, using the data for days per solar year and days per lunar month, calculate the number of months in the solar cycle for each (column B). One then calculates the difference between that figure and the closest whole number and compares them to the annualized difference for 19 years/234.997028544589 months (0.002971455411114 of a lunar month divided by 19, or 6.65 minutes per year) (column C). **It turns out that no truly better solution appears until you reach the pair of 334 years and 4131 months** (4131.00039652067 months is only 0.00039652066789 of a lunar month longer in 334 years, or 3.029 seconds per solar year). No fundamentally better solutions, minimizing the amount of time gained or lost per year by following a lunar cycle, exist between 19 and 334.

[18] 6.65 minutes times 216 years equals 1436 minutes, which is 23 hours and 56 minutes.

[19] See note 1.

Chart 1

A	B	C	Column C expressed in seconds
1	12.36826466024150	0.36826466024152	939,606.400
19	234.99702854458900	0.00015639239006	399.026
20	247.36529320483000	0.01826466024152	46,601.245
284	3512.58716350859000	0.00145364961763	3,708.905
296	3661.00633943149000	0.00002141699828	54.644
315	3896.00336797608000	0.00001069198755	27.280
334	4131.00039652067000	0.00000118718763	3.029
349	4316.52436642429000	0.00136284692180	3,477.227
945	11688.01010392820000	0.00001069198755	27.280
998	12343.52813092100000	0.00047281470838	1,206.360
1002	12393.00118956200000	0.00000118718763	3.029

Thus, to design a calendar to nearly eliminate the problem of the 6.65 minute discrepancy requires one design a calendar with a cycle of 334 years. Indeed, a 334 year calendar would only gain 3.029 seconds per year, or one day every 28,524 years. But such a calendar would be much less workable, as it requires the distribution of the massive amounts of non-repeating data **(a matrix 334 times at least 10; more than 3340 data points[20]).** A cycle of 334 years would also require adding 123 leap years in that time.[21] One could not even readily create as simple a device as a mnemonic to remember the occurrence of leap years in a 334 year cycle. Such complexities would fundamentally defeat the basic purpose of the calendar for a Diaspora-based Jewry lacking central authority. Instead, the Jewish calendar opted for a simpler, if slightly less accurate, solution. The 19-year cycle of the Jewish calendar is so simple that in a couple of days, any arithmetically skilled person can create a perpetual calendar from its rules.[22] As Rambam states, "even young school children can master [the calculations] entirely in three or four days."[23]

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[20] Whether or not such a matrix could even be constructed is unclear. The 247-year matrix appears to repeat (over short intervals) because, although 19 year cycles vary in length from 6939 to 6942 days, thirteen 19-year cycles always contain 90,216 days (for a constant average of 6939.692308 days per cycle). Given the variability of the Molad time as well as the "postponements" built into the calendar, the patterns of years during those 90,216 days may repeat for two or three cycles, but not much longer. Similarly, though 334-year cycles would vary in length between 121,990 and 121,992 days, ten 3440-year cycles contain 1,219,909 days, or 121,909.1 days per cycle. The patterns of years do not seem, however, to repeat even once.

[21] This is derived by solving the following simultaneous equations: X + Y = 334 and 12X +13Y = 4341, where X represents the number of years with 12 months and Y represents the number of years with 13 months. Solving the equations gives values of 211 and 123 for X and Y, respectively.

[22] See *Mishnah Berurah*, supra note 14.

[23] *Kiddush Hachodesh* 11:4.

Fundamental Aims of the Jewish Calendar

An important question to ponder: why did the Talmudic Sages mandate the fixing of the Jewish calendar? Why could it not have remained visually established in Jerusalem each and every month, as it was for the first 1000 or 1500 years of Jewish life? Our discussion will address this issue, which one could call "the anchoring of Jewish practices."

First, one ought to consider the conditions that allowed for the successful promulgation of an ongoing calendar based on visual testimony and corroboration. For one, a sole, central location was needed both to facilitate the process and ensure the calendar's singularity. Second, the validation of testimony, proclamation of the months, and dissemination of the calendar required autonomous centralized leadership whose decisions were binding on and expedited by the Jewish community. Next, decisions made by that authority needed to reach the populace without distortion or delay. And finally, even given relative ease of communication, the area where the information needed to reach must have been necessarily limited.

Breakdown of one of these factors would significantly impact the perpetuation of a constantly generating calendar; breakdown of more could be disastrous. Yet that is precisely what the rabbis of Hillel II's generation foresaw continuing to happen. For hundreds of years since the destruction of the Temple, the seat of Jewish leadership had wandered. Harsh conditions in Israel proper continued to erode the power and efficacy of its Jewish authorities. Babylonia not only was home to the majority of the Jewish population; many of its centers had also surpassed the Palestinian Jewish community in terms of importance and leadership. Communication between the various Jewish settlements was not always expedient, and as time went on, the Jewish diaspora extended farther and farther from the centers of Palestine.

The Talmudic rabbis thus set out to create a fixed calendar independent of central location and authority. The rules, once learned, were simple enough for anyone to apply. One could travel to a far-off land and, with but a few data points, recreate the calendar. More likely, one would set off with one small matrix of 19 by 13 data points in one's pocket containing fixed patterns for the cycle of years (and perhaps one other chart for Torah reading). Not only would a new community not need to rely on the pronouncements of a central authority such as the Sanhedrin (high court)—it need not even communicate with such a body, nor would such a body even need to exist! The central aim was for communities and individuals to be easily able to maintain a Jewish calendar in perpetuity. *It is indeed remarkable that to this day, all Jews, regardless of denominational affiliation, share this single Jewish calendar established so many years ago.*

Given these goals, one has an even greater appreciation of the elegance and relative simplicity of the system the Rabbis developed. Jewish law posited that the lunar month was 29 days, 12 hours, 44 minutes and 3 1/3 seconds long. The Jewish calendar also seemingly presumed that the solar year was 365 days, five hours, 55 minutes and 25/57 seconds long. This allowed the cycle of 19/235 to work out exactly.

At first blush, the rabbis erred, as this data, especially the length of the solar year, is significantly incorrect. The actual mean solar year is 365 days, five hours, 48 minutes, and 46 seconds long. Using the 19-year/235-month cycle generates a disparity of 6 2/3 minutes per year, which is one day every 216 years.

If only the Talmudic rabbis had had the right data, this problem would not have happened, one laments. It turns out that that lament—often repeated in many different forms of literature—is wrong. Whether one uses the Talmudic data or

modern astronomical data,[24] once comes to the conclusion that if one wishes to use a simple luni-solar calendar, one adopts the 19-year cycle as the framework and the error that creeps in cannot be fixed without disrupting the basic nature of the automatic cycle, and thus making it extremely cumbersome to use.

Long-term Solutions to the Calendar Difficulties

Before concluding this article, it is worthwhile to consider if, given the 6-2/3-minute annual difference between the Jewish and solar calendars, leading to a growing disparity between the two, any remedies do in fact exist.

Most sources dealing with our topic, medieval and modern, generally close with the same sentiment: "Yes, there is a difference between the actual solar cycle and the Jewish calendar. We are to acknowledge this (it is often expressed) as a sign of our faith in an ultimate redemption, in which the mathematical machinations of the fixed calendar will be replaced. At that time, we will return to the system of observation, testimony, and pronouncement of the months by the Sanhedrin. All will then, presumably, be repaired."

But what exactly would the Sanhedrin do to remedy the situation? Presumably, one concludes, they would take a lunar month which otherwise would have been 30 days long, declare it to be 29 days, and thus have the Jewish calendar "catch up" to the solar cycle. This would have to be repeated often enough to make up for the difference of one day per 216 years that the calendar has been in existence.

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[24] Even if one uses the calculations of Shmuel, that the solar year was exactly 365 days and six hours long, no fundamentally better cycle than 19 years/235 months exists until one reaches 483 solar years/5974 lunar months. (Even though 255,264, 283, 302, 321, 350, 369, 388, 407, 426, 445, and 464 seem better, an analysis similar to that in note 5 applies as well.)

Would this solution in fact work? The answer is a resounding no. Remember, the Jewish calendar is first and foremost a lunar calendar—the months, the calendar's basic building blocks, are designed to correlate to the phases of the moon. During the 216-year period in which the year of the Jewish calendar outpaces the solar cycle by about one day, the months of the Jewish calendar are still aligned with the true Molad.[25] Adjusting the length of a month would fulfill the secondary goal of the Jewish calendar, namely to keep the lunar cycles in close correspondence to the solar years, but at what cost? Shifting even one or two days would break the calendar's faithful connection of Rosh Chodesh to the Molad date.[26] Adding many days could cause the new moon and Rosh Chodesh to be distanced by a week or more! Thus, there is no ready remedy to this problem.

A true solution—assuming the Sanhedrin even has an interest in maintaining a fixed calendar—along these lines is indeed workable, but it would take much more time. As the disparity between the solar and lunar cycles approached 29 or 30 days, the Sanhedrin would take a year that otherwise would be a leap year and declare it to be a regular year—thus eliminating 30 added days.[27] In that way, the annual cycles (years) constructed from the

[25] More or less—the delays added to the calendar at the beginning of a new year temporarily distance Rosh Hashanah from the Molad, but the difference quickly catches up.

[26] Even though there are other instances of Rosh Chodesh observed at a slight distance from the actual Molad (as mentioned above), in all those cases, the Molad occurs before Rosh Chodesh. This change would invert that pattern, such that Rosh Chodesh would consistently precede the Molad!

[27] Alternatively, the month could be deducted after the difference between the solar and lunar cycles exceeded 15 days. The calendar would then shift from being 15 days ahead to 15 days behind, and resume its tendency to gain time. This would have the "temporary" effect of distancing Rosh Chodesh from the actual Molad date. This solution is unworkable, however, as it would cause Pesach to occasionally occur before Tekufat Nisan begins, in violation of the halachic rules which necessitated a luni-solar calendar in the first

lunar approximations would realign with the actual solar cycle, while the months would not deviate from the occurrence of the Molad.

For those who worry about this problem, such a solution cannot be implemented until some time after the year 6600 CE (around the year 10,360 in the Jewish calendar),[28] in more than 4,000 years! Moreover, even with this growing discrepancy, the Jewish calendar designed by Hillel II does not truly cease functioning until the year 15,115 CE, (18,875 in the Jewish calendar)—some 13,000 years from now,[29] when Pesach falls out after Tekufat Tammuz (the summer solstice, approximately June 21)—in the summer rather than in the spring.[30]

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place.

[28] 216 years times 29 equals 6,264 years. If the Jewish calendar was instituted around 350 CE, our calendar will have gained 29 days around the year 6614. This is exactly the error that would have been eliminated by the 334 year calendar. In 19 sets of 334 (6,346) years, there would have been 2,337 (19 sets of 123 per cycle) additional leap months, instead of the 2,338 (334 sets of 7 per cycle) leap months over the same period of time in the calendar we use.

[29] Pesach first begins to occur in June in 10,517 CE. In the year 13,747 CE, Pesach occurs on June 15, causing the seventh day of Pesach to occur on June 21. In 15,115, Pesach will begin on June 22. As noted above, the solar years are speeding up slightly, thus the solstices may occur slightly earlier, so that the Jewish calendar may fail somewhat before these dates.

[30] There is a dispute as to whether Pesach must be correlated with spring such that it must occur within 30 days of Tekufat Nisan, or merely sometime between Tekufat Nisan and Tekufat Tammuz. Rashi, BT *Sanhedrin* 13b and *Rosh Hashanah* 21a, explains the obligation to keep Passover in "*Aviv*" as a requirement to add a leap month if the spring solstice were to occur later than the first half of the lunar month of Nisan. By implication, the spring solstice can be no earlier than 30 days before Pesach (in such a case, there would be no need to add a leap month). Rambam (*Kiddush Hachodesh* 4:1), however, defines the obligation as simply the need to add periodic leap months so that Pesach not occur in the winter or the summer.

Since the year 1758 CE, Pesach has periodically occurred on or after April 22, more than 30 days after the vernal equinox. The structure of our calendar thus clearly follows Rambam's opinion;

For those who think this article incomplete without proposing a solution, the ready solution is to skip the leap month scheduled in 10,336 of the Jewish calendar (6576 CE). That year is *kesidran* (Cheshvan 29 days, Kislev 30 days) and year 19 of the 544th 19-year cycle since the creation year, which makes skipping the leap month a bit simpler.[31] Implementation of this solution would require the assent of all significant segments of the Jewish community, something which we hope will be achievable at that time. In the meantime, Jewish tradition and Jewish life have many more pressing, short-term problems on which great minds ought to focus. Sometimes, it is not worth one's while to invest too much in solving problems that are very distant in the future. This is even more true when the price to pay for fixing the problem may very well turn out to be further fracturing of communal unity.

Conclusion

An analytic review is worthwhile to recap the problems and possible solutions facing any attempt to manage a luni-solar calendar.

A luni-solar calendar is more complicated than either a pure solar or pure lunar calendar, in that it attempts, over time, to keep lunar months correlated with solar years. The difficulty arises because a lunar month is 29.53059 days long while a solar year is 365.24219 days long. Consequently, the number of months does not divide evenly into a single solar year. In order to keep a luni-solar calendar with no partial months, the

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therefore, the breakdown point will occur only when Pesach strays from the season between the vernal equinox and summer solstice. This misunderstanding has led some to mistakenly call for immediate calendar reform in order to ensure that Pesach always occurs within 30 days of the equinox. (See, e.g., W. M. Feldman, *Rabbinical Mathematics and Astronomy*, 3rd ed. (London: Herman Press, 1978), 207-08.)

[31] This data was prepared with the assistance of www.kaluach.com.

calendar must adopt a multi-year cycle with leap months added periodically. The Jewish calendar does this by adding seven leap months in 19 years for a total of 235 months.

The true astronomical length of the solar year (365.24219 days) differs from the presumed length of the solar year in the Jewish calendar (365.246822 days). In reality, there is no value of an average solar year that is input into the Jewish calendar. It is merely the result of building the calendar primarily around the lunar cycles and only then to bring the calendar into close approximation of the solar year as well. As such, the length of the year usually attributed to Rav Adda is a fictitious number, derived from 1/19 of 235 lunar months. Nonetheless, once the calendar is based on the 19-year/235-month cycle, the Jewish calendar continues to advance 6.65 minutes ahead of each solar year, one day ahead every 216 years.

One theoretical solution, we have seen, is to continue the pre-Talmudic system of creating a calendar on an ongoing basis through visual calculations through the high court (Sanhedrin) in Jerusalem. This would not succeed practically, as a great deal of communal unity and hierarchical authority is needed to keep, and run, a regular calendar of this type. Undoubtedly some groups will disrespect this calendar produced by this rabbinical body, further fracturing the already divided Jewish people. Indeed, it is worth noting that even as (in our modern times) there is little consensus about anything across the Jewish spectrum, all Jews still follow the same calendar—a tribute to the wisdom of a fixed calendar.

A second solution is to create a much larger calendar cycle to address this problem. To run a much better system than one that loses a day every 216 years, one would have to adopt a calendar that functions on a 334-year cycle, which takes pages and pages to lay out and is very hard to use (as it has much more complicated rules) and is a non-repeating cycle of 4,131 lunar months. Our

current 19-year cyclical calendar, in contrast, can be expressed as a matrix of 19 by 13 for a total of 247 data points (as found in the *Tur*[32]); a 334-year calendar would need a matrix of 334 by (at least) 10, for a minimum of 3340 data points). Such a cumbersome calendar would have failed to serve its purpose of creating a simple, predictable, and usable calendar to unite the Jewish people.

A third solution is to adopt an excellent approximation as a way of balancing the need for unity and uniformity (through a fixed calendar, independent of central authority residing in a fixed location) with ease of use and simplicity. Even if one attempts to preserve the integrity of the true astronomical length of the solar year, one readily concludes that the best manageable solution is to incorporate the close approximation already used by the Jewish tradition. In doing so, one develops a 19-year calendar faithful to both the lunar and solar cycles, but which gains some 6.65 minutes per year—one day every 216 years. Thus, one has no choice but to accept that our current calendar is very, very accurate, but not perfect, and will not create significant problems (such as Pesach occurring in the summer) until past the year 15,115 CE. On the whole, this solution seems reasonable, and designed to accomplish its basic aims.

Postscript

The Jewish calendar has much to teach us about the coexistence of multiple truths, and the value of unity. The universe seems to present us with two different calendar truths, two disparate realities: solar truth of years and lunar truth of months. These realities as they appear to us are not readily compatible—as systems of measurement they seem to

[32] O.C. 428.

stand at odds with one another; one apparently excludes the other. Can people live with such tension?

The Islamic calendar makes a choice. By creating an entirely lunar calendar, it sacrifices solar truth for lunar truth—its years do not correspond to the solar seasons at all. The Julian and Gregorian calendars endorsed by Christian leadership choose as well. Their wholly solar calendars sacrifice lunar truth for solar truth—months bear no relationship to the lunar months.

Halacha, however, demands something else. The Jewish calendar struggles to defend both truths and yet be a system that is both workable and creates a common calendar, which is needed to have a common community. The luni-solar calendar we have had for hundreds and hundreds of years strives to bring solar truth and lunar truth into closer alignment. Our Jewish calendar is but another example of our general understanding of the forces at work in the Jewish tradition when searching for truth. Halacha often highlights a dialectic tension—two divergent truths in stark opposition. Each is a value unto itself; each ought not simply be cast off in favor of the other. Instead Halacha specifically demands that both truths be acknowledged and reckoned with. Truth is found in the middle ground; it is complex and, at first blush, inconsistent. Only after much study does its beauty appear. In the case of our calendar, Halacha adopts a complex approach, involving mathematical manipulation, in order to maintain a correlation between lunar months and solar seasons. Ultimately, shades of gray are more elegant than either all black or all white. The Jewish calendar thus reminds us of our higher calling—to work diligently so that multiple truths can, if not blend together, at least continue to stand at tension with one another in our lives.

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